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CHERN–SIMONS TERM AT FINITE DENSITY AND TEMPERATURE

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The Chern–Simons topological term dynamical generation in the effective action is obtained at arbitrary finite density and temperature. By using the proper time method and perturbation theory it is shown that at zero temperature $\mu^2 = m^2$ is the crucial point for Chern–Simons term. So when $\mu^2 < m^2$, μ influence disappears and we get the usual Chern–Simons term. On the other hand, when $\mu^2 > m^2$, the Chern–Simons term vanishes because of nonzero density of background fermions. In particular for massless case parity anomaly is absent at any finite density or temperature. This result holds in any odd dimension both in Abelian and in non-Abelian cases.

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Динамическая генерация черн-саймоновского члена при конечной плотности и температуре

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Получен коэффициент при черн-саймоновском члене в эффективном действии при произвольной плотности и температуре. При использовании метода собственного времени и теории возмущения показано, что $\mu^2 = m^2$ является критической точкой для члена Черн-Саймонса при нулевой температуре. Так, при $\mu^2 < m^2$ μ-зависимость исчезает и получается обычный черн-саймоновский член. При $\mu^2 > m^2$ черн-саймоновский член исчезает из-за ненулевой плотности фоновых электронов. В частности, для безмассового случая аномалия четности отсутствует при любой ненулевой плотности или температуре. Полученный результат справедлив в любой нечетномерной размерности как для абелева, так и для неабелева случая.

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Since introduction of the Chern–Simons (CS) topological term [1] and by now, the great number of papers devoted to it appeared. Such interest is explained by variety of significant physical effects caused by CS secondary characteristic class. These are, for example, gauge particles mass appearance in quantum field theory, applications to condense

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matter physics such as the fractional quantum Hall effect and high T_c superconductivity, possibility of free of metric tensor theory construction and so on.

It was shown [2—4] in a conventional zero density gauge theory, that the CS term is generated in the Eulier–Heisenberg effective action by quantum corrections. The main goal of this paper is to explore the parity anomalous CS term generation at finite density. In the excellent paper by Niemi [5] it was emphasized that the charge density at $\mu \neq 0$ becomes a nontopological object, i.e., contains as topological part so as nontopological one. The charge density at $\mu \neq 0$ (nontopological, neither parity-odd nor parity-even object)* in QED_3 at finite density contains as well parity-odd part corresponding to CS term so as parity even part, which cannot be covariantized and does not contribute to the mass of the gauge field. Here we are interested in effect of finite density influence on covariant parity-odd form in action leading to the gauge field mass generation — CS topological term. Deep insight on this phenomena at small densitied was done in [5,7]. The result for CS term coefficient in QED_3 is $\left[\operatorname{th}\frac{1}{2}\beta(m-\mu)+\operatorname{th}\frac{1}{2}\beta(m+\mu)\right]$ (see [7], formulas (11.18)). However, to get this result it was heuristically supposed that at small densities index theorem could still be used and only odd in energy part of spectral density is responsible for parity nonconserving effect. Because of this in [7] it had been stressed that the result holds only for small μ. However, as we'll see below this result holds for any values of chemical potential. Thus, to obtain trustful result at any values of μ one has to use transparent and free of any restrictions on μ procedure, which would allow to perform calculations with arbitrary non-Abelian background gauge fields.

Since the chemical potential term $\mu\bar{\psi}\gamma^0\psi$ is odd under charge conjugation we can expect that it would contribute to P and CP nonconserving quantity — CS term. As we will see, this expectation is completely justified.

The zero density approach usually is a good quantum field approximation when the chemical potential is small as compared with characteristic energy scale of physical processes. Nevertheless, for investigation of topological effects it is not the case. As we will see below, even a small density could lead to principal effects.

Introduction of a chemical potential μ in a theory corresponds to the presence of a nonvanishing background charge density. So, if $\mu > 0$, then the number of particles exceeds that of antiparticles and vice versa. It must be emphasized that the formal addition of a chemical potential looks like a simple gauge transformation with the gauge function μt. However, it doesn't only shift the time component of a vector potential but also gives corresponding prescription for handling Green's function poles. The correct introduction of a chemical potential redefines the ground state (Fermi energy), which leads to a new spinor propagator with the correct ε-prescription for poles. So, for the free spinor propagator we have (see, for example, [8,9])

$$G(p;\mu)=\frac{\tilde{\not p}+m}{(\tilde{p}_0+i\varepsilon\,\mathrm{sgn}\,p_0)^2-\mathbf{p}^2-m^2}, \tag{1}$$

*For abbreviation, speaking about parity invariance properties of local objects, we will keep in mind symmetries of the corresponding action parts.

where $\tilde{p} = (p_0 + \mu, \mathbf{p})$. Thus, when $\mu = 0$, one at once gets the usual ε-prescription because of the positivity of $p_0 \operatorname{sgn} p_0$. In the presence of a background Yang–Mills field we consequently have for the Green function operator

$$\hat{G} = (\gamma\tilde{\pi} - m) \frac{1}{(\gamma\tilde{\pi})^2 - m^2 + i\varepsilon(p_0 + \mu) \operatorname{sgn}(p_0)}, \tag{2}$$

where $\tilde{\pi}_\nu = \pi_\nu + \mu\delta_{\nu 0}$; $\pi_\nu = p_\nu - gA_\nu(x)$.

Let's first consider a (2 + 1) dimensional Abelian case and choose the background field in the form

$$A^\mu = \frac{1}{2} x_\nu F^{\nu\mu}, \quad F^{\nu\mu} = \text{Const.}$$

To obtain the CS term in this case, it is necessary to consider the background current

$$\langle J^\mu \rangle = \frac{\delta S_{\text{eff}}}{\delta A_\mu}$$

rather than the effective action itself. This is because the CS term formally vanishes for such the choice of A^μ but its variation with respect to A^μ produces a nonvanishing current. So, consider

$$\langle J^\mu \rangle = -ig \operatorname{tr} [\gamma^\mu G(x, x')]_{x \to x'}, \tag{3}$$

where

$$G(x, x') = \exp\left(-ig \int_{x'}^{x} d\zeta_\mu A^\mu(\zeta) \right) \langle x | \hat{G} | x' \rangle. \tag{4}$$

Let's rewrite Green function (2) in a more appropriate form

$$\hat{G} = (\gamma\tilde{\pi} - m) \left[\frac{\theta((p_0 + \mu) \operatorname{sgn}(p_0))}{(\gamma\tilde{\pi})^2 - m^2 + i\varepsilon} + \frac{\theta(-(p_0 + \mu) \operatorname{sgn}(p_0))}{(\gamma\tilde{\pi})^2 - m^2 - i\varepsilon} \right]. \tag{5}$$

Now, we use the well-known integral representation of denominators

$$\frac{1}{\alpha \pm i0} = \mp i \int_0^\infty ds\, e^{\pm i\alpha s},$$

which corresponds to introducing the «proper-time» s into the calculation of the Eulier–Heisenberg Lagrangian by the Schwinger method [10]. We obtain

$$\hat{G} = (\gamma\tilde{\pi} - m) \left[-i \int_0^\infty ds \exp\left(is[(\gamma\tilde{\pi})^2 - m^2 + i\varepsilon]\right) \theta((p_0 + \mu) \operatorname{sgn}(p_0)) + \right.$$

$$\left. + i \int_0^\infty ds \exp\left(-is[(\gamma\tilde{\pi})^2 - m^2 - i\varepsilon]\right) \theta(-(p_0 + \mu) \operatorname{sgn}(p_0)) \right]. \tag{6}$$

For simplicity, we restrict ourselves only to the magnetic field case, where $A_0 = 0$, $[\tilde{\pi}_0, \tilde{\pi}_\mu] = 0$. Then we easily can factorize the time dependent part of Green function

$$G(x, x') = \int \frac{d^3p}{(2\pi)^3} \hat{G}\, e^{ip(x-x')} = \int \frac{d^2p}{(2\pi)^2} \hat{G}_{\mathbf{x}}\, e^{i\mathbf{p}(\mathbf{x}-\mathbf{x}')} \int \frac{dp_0}{2\pi} \hat{G}_{x_0}\, e^{ip_0(x_0 - x'_0)}. \tag{7}$$

By using the obvious relation

$$(\gamma\tilde{\pi})^2 = (p_0 + \mu)^2 - \pi^2 + \frac{1}{2} g\sigma_{\mu\nu}F^{\mu\nu} \tag{8}$$

one gets

$$G(x, x')\,|_{x \to x'} = -i \int \frac{dp_0}{2\pi} \frac{d^2p}{(2\pi)^2} (\gamma\tilde{\pi} - m) \int_0^\infty ds \Big[e^{is(\tilde{p}_0^2 - m^2)}\, e^{-is\tilde{\pi}^2}\, e^{isg\sigma F/2} -$$

$$- \theta(-(p_0 + \mu)\,\mathrm{sgn}\,(p_0)) \Big(e^{is(\tilde{p}_0^2 - m^2)}\, e^{-is\pi^2}\, e^{isg\sigma F/2} + e^{-is(\tilde{p}_0^2 - m^2)}\, e^{is\pi^2}\, e^{-isg\sigma F/2} \Big) \Big]. \tag{9}$$

Here the first term corresponds to the usual μ-independent case and there are two additional μ-dependent terms. In the calculation of the current the following trace arises:

$$\mathrm{tr}\,[\gamma^\mu(\gamma\tilde{\pi} - m)\, e^{isg\sigma F/2}] =$$

$$= 2\pi^\nu g^{\nu\mu} \cos(g\,|{}^*F|\,s) + 2\frac{\pi^\nu F^{\mu\nu}}{|{}^*F|} \sin(g\,|{}^*F|\,s) - 2im\frac{{}^*F^\mu}{|{}^*F|} \sin(g\,|{}^*F|\,s),$$

where ${}^*F^\mu = \varepsilon^{\mu\alpha\beta}F_{\alpha\beta}/2$ and $|{}^*F| = \sqrt{B^2 - E^2}$. Since we are interested in calculation of the parity odd part (CS term) it is enough to consider only terms proportional to the dual strength tensor ${}^*F^\mu$. On the other hand, the term $2\pi^\nu g^{\nu\mu}\cos(g\,|{}^*F|\,s)$ at $\nu = 0$ (see expression for the trace, we take in mind that here there is only magnetic field) also gives nonzero contribution to the current $J^0_{c.s.}$ [6].

$$J^0_{\text{even}} = \frac{|eB|}{2\pi} \left(\mathrm{Int}\left[\frac{\mu^2 - m^2}{2\,|eB|} \right] + \frac{1}{2} \right) \theta(|\mu| - |m|). \tag{10}$$

This part of current is parity invariant because under parity $B \to -B$. It is clear that this parity-even object does contribute neither to the parity anomaly nor to the mass of the gauge field. Moreover, this term has been obtained [6] in the pure magnetic background and scalar magnetic field occurs in the argument's denominator of the cumbersome function — integer part. So, the parity even term seems to be «noncovariantizable», i.e., it cannot be converted in covariant form in effective action. For a parity, in papers [6] charge density consisting of both parity-odd and parity-even parts is dubbed CS, what leads to misunderstanding. The main goal of this paper is to explore the parity anomalous topological CS term in the effective action at finite density. So, just the term proportional to the dual strength tensor ${}^*F^\mu$ will be considered. The relevant part of the current reads

$$J^\mu_{\text{CS}} = \frac{g}{2\pi} \int dp_0 \int \frac{d^2p}{(2\pi)^2} \int_0^\infty ds\, \frac{2im\,{}^*F^\mu}{|{}^*F|} \sin(g\,|{}^*F|\,s) \Big[e^{is(\tilde{p}_0^2 - m^2)}\, e^{-is\pi^2} -$$

$$- \theta(-(p_0 + \mu) \operatorname{sgn}(p_0))\left(e^{is(\tilde{p}_0^2 - m^2)} e^{-is\pi^2} - e^{-is(\tilde{p}_0^2 - m^2)} e^{is\pi^2}\right)\Bigg]. \tag{11}$$

Evaluating two-momentum integral we derive

$$J^{\mu}_{CS} = \frac{g^2}{4\pi^2} m^* F^{\mu} \int_{-\infty}^{+\infty} dp_0 \int_0^{\infty} ds \left[e^{is(\tilde{p}_0^2 - m^2)} - \theta(-\tilde{p}_0 \operatorname{sgn}(p_0))\left(e^{is(\tilde{p}_0^2 - m^2)} + e^{-is(\tilde{p}_0^2 - m^2)}\right)\right]. \tag{12}$$

Thus, we get besides the usual CS part [3], also the μ-dependent one. It is easy to calculate it by the use of the formula

$$\int_0^{\infty} ds\, e^{is(x^2 - m^2)} = \pi \left(\delta(x^2 - m^2) + \frac{i}{\pi} \mathcal{P} \frac{1}{x^2 - m^2} \right)$$

and we get eventually

$$J^{\mu}_{CS} = \frac{m}{|m|} \frac{g^2}{4\pi} {}^*F^{\mu} [1 - \theta(-(m + \mu) \operatorname{sgn}(m)) - \theta(-(m - \mu) \operatorname{sgn}(m))] =$$

$$= \frac{m}{|m|} \theta(m^2 - \mu^2) \frac{g^2}{4\pi} {}^*F^{\mu}. \tag{13}$$

Let's now discuss the non-Abelian case. Then $A^{\mu} = T_a A^{\mu}_a$ in (2) and

$$\langle J^{\mu}_a \rangle = -ig \operatorname{tr}\left[\gamma^{\mu} T_a G(x, x')\right]_{x \to x'}.$$

It is well known [3,11] that there exist only two types of the constant background fields. The first is the «Abelian» type (it is easy to see that the self-interaction $f^{abc} A^{\mu}_b A^{\mu}_c$ disappears under that choice of the background field)

$$A^{\mu}_a = \eta_a \frac{1}{2} x_{\nu} F^{\nu\mu}, \tag{14}$$

where η_a is an arbitrary constant vector in the color space, $F^{\nu\mu} = \text{Const}$. The second is the pure «non-Abelian» type

$$A^{\mu} = \text{Const}. \tag{15}$$

Here the derivative terms (Abelian part) vanish from the strength tensor and it contains only the self-interaction part $F^{\mu\nu}_a = g f^{abc} A^{\mu}_b A^{\mu}_c$. It is clear that to catch Abelian part of the CS term we should consider the background field (14), whereas for the non-Abelian (derivative noncontaining, cubic in A) part we have to use the case (15).

Calculations in the «Abelian» case reduce to the previous analysis, except the trivial adding of the color indices in formula (13):

$$J^{\mu}_a = \frac{m}{|m|} \theta(m^2 - \mu^2) \frac{g^2}{4\pi} {}^*F^{\mu}_a. \tag{16}$$

In the case (15) all calculations are similar. The only difference is that the origin of term $\sigma_{\mu\nu}F^{\mu\nu}$ in (8) is not the linearity A in x (as in Abelian case) but the pure non-Abelian $A^{\mu} = \text{Const}$. Here term $\sigma_{\mu\nu}F^{\mu\nu}$ in (8) becomes quadratic in A and we have

$$J^{\mu}_{a} = \frac{m}{|m|}\,\theta(m^2 - \mu^2)\,\frac{g^3}{4\pi}\,\varepsilon^{\mu\alpha\beta}\,\mathrm{tr}\,[T_a A^{\alpha} A^{\beta}]. \qquad (17)$$

Combining formulas (16) and (17) and integrating over field A^{μ}_{a} we obtain eventually

$$S^{\text{C.S.}}_{\text{eff}} = \frac{m}{|m|}\,\theta(m^2 - \mu^2)\pi W[A], \qquad (18)$$

where $W[A]$ is the CS term

$$W[A] = \frac{g^2}{8\pi^2}\int d^3x\varepsilon^{\mu\nu\alpha}\,\mathrm{tr}\left(F_{\mu\nu}A_{\alpha} - \frac{2}{3}gA_{\mu}A_{\nu}A_{\alpha}\right).$$

This result can be obtained also with an arbitrary initial field configuration by the use of the perturbative expansion. Here we work at once in the non-Abelian case.

Let's first consider non-Abelian 3-dimensional gauge theory. The only graphs whose P-odd parts contribute to the parity anomalous CS term are shown in Fig.1.

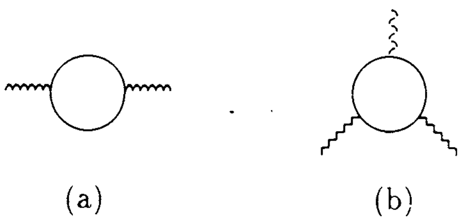

Fig.1. Graphs whose P-odd parts contribute to the CS term in non-Abelian 3D gauge theory

So, the part of effective action containing the CS term looks as

$$I^{\text{C.S.}}_{\text{eff}} = \frac{1}{2}\int_x A_{\mu}(x)\int_p e^{-ixp}A_{\nu}(p)\Pi^{\mu\nu}(p) +$$

$$+\frac{1}{3}\int_x A_{\mu}(x)\int_{p,r} e^{-ix(p+r)}A_{\nu}(p)A_{\alpha}(r)\Pi^{\mu\nu\alpha}(p, r), \qquad (19)$$

where polarization operator and vertices have a standard form

$$\Pi^{\mu\nu}(p) = g^2\int_k \mathrm{tr}\,[\gamma^{\mu}S(p+k;\mu)\gamma^{\nu}S(k;\mu)]$$

$$\Pi^{\mu\nu\alpha}(p, r) = g^3\int_k \mathrm{tr}\,[\gamma^{\mu}S(p+r+k;\mu)\gamma^{\nu}S(r+k;\mu)\gamma^{\alpha}S(k;\mu)], \qquad (20)$$

here the following notation is used $\int_x = \int_0^{\beta} dx_0 \int d\mathbf{x}$ and $\int_k = \frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int \frac{d\mathbf{k}}{(2\pi)^2}$. First consider the second order term (Fig.1, graph (a)). It is well known that the only object giving us the possibility to construct P and T odd form in action is Levi–Chivita tensor*. Thus, we will drop all terms noncontaining Levi–Chivita tensor. Signal for the mass generation (CS term) is $\Pi^{\mu\nu}(p^2=0)\neq 0$. So we get

$$\Pi^{\mu\nu}=g^2\int\limits_k(-i2m\varepsilon^{\mu\nu\alpha}p_\alpha)\frac{1}{(\tilde{k}^2+m^2)^2}. \tag{21}$$

After some simple algebra one obtains

$$\Pi^{\mu\nu}=-i2mg^2\varepsilon^{\mu\nu\alpha}p_\alpha\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{1}{(\tilde{k}^2+m^2)^2}=-i2mg^2\varepsilon^{\mu\nu\alpha}p_\alpha\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\frac{i}{4\pi}\frac{1}{\omega_n^2+m^2}, \tag{22}$$

where $\omega_n=(2n+1)\pi/\beta+i\mu$. Performing summation we get

$$\Pi^{\mu\nu}=i\frac{g^2}{4\pi}\varepsilon^{\mu\nu\alpha}p_\alpha\,\mathrm{th}\,(\beta m)\frac{1}{1+\mathrm{ch}\,(\beta m)/\mathrm{ch}\,(\beta m)}. \tag{23}$$

It is easily seen that at $\beta\to\infty$ limit we'll get zero temperature result [12]

$$\Pi^{\mu\nu}=i\frac{m}{|m|}\frac{g^2}{4\pi}\varepsilon^{\mu\nu\alpha}p_\alpha\theta(m^2-\mu^2). \tag{24}$$

In the same manner handling the third order contribution (Fig.1*b*) one gets

$$\Pi^{\mu\nu\alpha}=-2g^3i\varepsilon^{\mu\nu\alpha}\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{m(\tilde{k}^2+m^2)}{(\tilde{k}^2+m^2)^3}=$$

$$=-i2mg^3\varepsilon^{\mu\nu\alpha}\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{1}{(\tilde{k}^2+m^2)^2}, \tag{25}$$

and further all calculations are identical to the second order

$$\Pi^{\mu\nu\alpha}=i\frac{g^3}{4\pi}\varepsilon^{\mu\nu\alpha}\,\mathrm{th}\,(\beta m)\frac{1}{1+\mathrm{ch}\,(\beta m)/\mathrm{ch}\,(\beta m)}. \tag{26}$$

Substituting (23), (26) in the effective action (19) we get eventually

$$I_{\mathrm{eff}}^{\mathrm{C.S.}}=\mathrm{th}\,(\beta m)\frac{1}{1+\mathrm{ch}\,(\beta m)/\mathrm{ch}\,(\beta m)}\frac{g^2}{8\pi}\int d^3x\varepsilon^{\mu\nu\alpha}\,\mathrm{rt}\left(A_\mu\partial_\nu A_\alpha-\frac{2}{3}gA_\mu A_\nu A_\alpha\right). \tag{27}$$

Thus, we get CS term with temperature and density dependent coefficient.

Let's now consider 5-dimensional gauge theory. Here the Levi–Chivita tensor is 5-dimensional $\varepsilon^{\mu\nu\alpha\beta\gamma}$ and the relevant graphs are shown in Fig.2.

The part of the effective action containing CS term reads

$$I_{\mathrm{eff}}^{\mathrm{C.S.}}=\frac{1}{3}\int\limits_x A_\mu(x)\int\limits_{p,r}e^{-ix(p+r)}A_\nu(p)A_\alpha(r)\Pi^{\mu\nu\alpha}(p,r)+$$

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*In three dimensions it arises as a trace of three γ matrices (Pauli matrices).

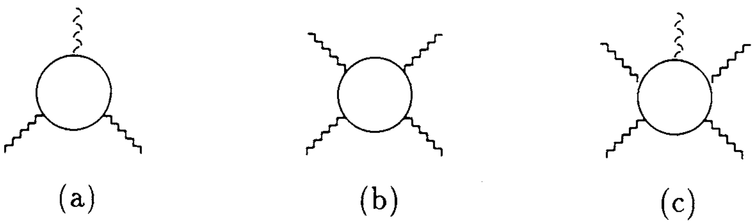

Fig.2. Graphs whose P-odd parts contribute to the CS term in non-Abelian 5D theory

$$+\frac{1}{4}\int_x A_\mu(x)\int_{p,r} e^{-ix(p+r+s)}A_\nu(p)A_\alpha(r)A_\beta(s)\Pi^{\mu\nu\alpha\beta}(p,r,s)+ \tag{28}$$

$$+\frac{1}{5}\int_x A_\mu(x)\int_{p,r} e^{-ix(p+r+s+q)}A_\nu(p)A_\alpha(r)A_\beta(s)A_\gamma(q)\,\Pi^{\mu\nu\alpha\beta\gamma}(p,r,s,q).$$

All calculations are similar to 3-dimensional case. First consider third order contribution (Fig.2*a*).

$$\Pi^{\mu\nu\alpha}(p,r)=g^3\int_k \mathrm{tr}\,[\gamma^\mu S(p+r+k;\mu)\gamma^\nu S(r+k;\mu)\gamma^\alpha S(k;\mu)]. \tag{29}$$

Taking into account that trace of five γ matrices in 5-dimensions is

$$\mathrm{tr}\,[\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\gamma^\rho]=4i\varepsilon^{\mu\nu\alpha\beta\rho},$$

we extract the parity odd part of the vertices

$$\Pi^{\mu\nu\alpha}=g^3\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}(i4m\varepsilon^{\mu\nu\alpha\beta\sigma}p_\beta r_\sigma)\frac{1}{(\bar{k}^2+m^2)^3}, \tag{30}$$

or in more transparent way

$$\Pi^{\mu\nu\alpha}=i4mg^3\varepsilon^{\mu\nu\alpha\beta\sigma}p_\alpha r_\sigma\frac{i}{\beta}\sum_{n=-\infty}^{+\infty}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(\omega_n^2+\mathbf{k}^2+m^2)^3}=$$

$$=i4mg^3\varepsilon^{\mu\nu\alpha\beta\sigma}p_\alpha r_\sigma\frac{i}{\beta}\sum_{n=-\infty}^{+\infty}\frac{-i}{64\pi^2}\frac{1}{\omega_n^2+m^2}. \tag{31}$$

Evaluating summation one comes to

$$\Pi^{\mu\nu\alpha}=i\,\mathrm{th}\,(\beta m)\frac{1}{1+\mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)}\frac{g^3}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta\sigma}p_\alpha r_\sigma. \tag{32}$$

Operating graphs (*b*) and (*c*) (Fig.2) in the same way one will obtain

$$\Pi^{\mu\nu\alpha\beta}=i\,\mathrm{th}\,(\beta m)\frac{1}{1+\mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)}\frac{g^4}{8\pi^2}\varepsilon^{\mu\nu\alpha\beta\sigma}s_\sigma, \tag{33}$$

$$\Pi^{\mu\nu\alpha\beta\gamma} = i \,\mathrm{th}\,(\beta m) \frac{1}{1 + \mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)} \frac{g^5}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta\sigma}. \tag{34}$$

Substituting (32)—(34) in the effective action (28) we get the final result for CS in 5-dimensional theory

$$I^{\mathrm{C.S.}}_{\mathrm{eff}} = i \,\mathrm{th}\,(\beta m) \frac{1}{1 + \mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)} \frac{g^3}{48\pi^2} \int_x \varepsilon^{\mu\nu\alpha\beta\gamma} \times$$
$$\times \mathrm{tr}\left(A_\mu \partial_\nu A_\alpha \partial_\beta A_\gamma + \frac{3}{2} g A_\mu A_\nu A_\alpha \partial_\beta A_\gamma + \frac{3}{5} g^2 A_\mu A_\nu A_\alpha A_\beta A_\gamma \right). \tag{35}$$

It is remarkable that all parity odd contributions are finite as in 3-dimensional so as in 5-dimensional cases. Thus, all values in the effective action are renormalized in a standard way, i.e., the renormalizations are determined by conventional (parity even) parts of vertices.

From the above direct calculations it is clearly seen that the chemical potential and temperature dependent coefficient is the same for all parity-odd parts of diagrams and doesn't depend on space dimension. So, the influence of finite density and temperature on CS term generation is the same in any odd dimension:

$$I^{\mathrm{C.S.}}_{\mathrm{eff}} = \mathrm{th}\,(\beta m) \frac{1}{1 + \mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)} \pi W[A] \xrightarrow{\beta \to \infty} \frac{m}{|m|} \theta(m^2 - \mu^2)\pi W[A], \tag{36}$$

where $W[A]$ is the CS secondary characteristic class in any odd dimension. Since only the lowest orders of perturbative series contribute to CS term at finite density and temperature (the same situation is well known at zero density), the result obtained by using formally perturbative technique appears to be nonperturbative. Thus, the μ-dependent CS term coefficient reveals the amazing property of universality. Namely, it does not depend on either dimension of the theory or Abelian or non-Abelian gauge theory is studied.

The arbitrariness of μ gives us the possibility to see CS coefficient behaviour at any masses. It is very interesting that $\mu^2 = m^2$ is the crucial point for CS at zero temperature. Indeed, it is clearly seen from (36) that when $\mu^3 < m^2$, μ influence disappeaars and we get the usual CS term $I^{\mathrm{C.S.}}_{\mathrm{eff}} = \pi W[A]$. On the other hand, when $\mu^2 > m^2$, the situation is absolutely different. One can see that here the CS term disappears because of nonzero density of background fermions. We'd like to emphasize the important massless case $m = 0$ considered in [7]. Then even negligible density or temperature, which always take place in any physical processes, lead to vanishing of the parity anomaly. Let us stress again that we nowhere have used any restrictions on μ. Thus we not only confirm result in [7] for CS in QED^3 at small density, but also expand it on arbitrary μ, non-Abelian case and arbitrary odd dimension.

In conclusion we'd like to emphasize that nevertheless there is connection between chiral anomaly and CS term at zero density due to trace identities, at finite density this connection is loosed. That is because of different nature of these objects. The chiral anomaly is an effect of regularization, but the chemical potential doesn't introduce new divergence in a theory. So it doesn't effect on chiral anomaly. On the other hand, CS term is essentially an effect of the finite part of the theory. So as we've seen finite density and temperature plays a crucial role in CS term generation.

References

1. Jackiw R., Templeton S. — Phys. Rev., 1981, v.D23, p.2291.
2. Niemi A.J., Semenoff G.W. — Phys. Rev. Lett., 1983, v.51, p.2077.
3. Redlich A.N. — Phys. Rev., 1984, v.D29, p.2366.
4. Alvarez-Gaume L., Witten E. — Nucl. Phys., 1984, v.B234, p.269.
5. Niemi A.J. — Nucl. Phys., 1985, v.B251[FS13], p.155.
6. Lykken J.D., Sonnenschen J., Weiss N. — Phys. Rev., 1990, v.D42, p.2161; Schakel A.M.J. — Phys. Rev., 1991, v.D43, p.1428; Zeitlin V.Y. — Mod. Phys. Lett., 1993, v.A8, p.1821.
7. Niemi A.J., Semenoff G.W. — Phys. Rep., 1986, v.135, No.3, p.99.
8. Shuryak E.V. — Phys. Rep., 1980, v.61, p.73.
9. Chodos A., Everding K., Owen D.A. — Phys. Rev., 1990, v.D42, p.2881.
10. Schwinger J. — Phys. Rev., 1951, v.82, p.664.
11. Brown L.S., Weisberger W.I. — Nucl. Phys., 1979, v.B157, p.285.
12. Sissakian A.N., Shevchenko O.Yu., Solganik S.B. — Phys. Lett., 1997, v.B403, p.75.

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